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Generalized Coulomb gap in the spherical version of a lattice model of disordered and correlated localized particles

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We study the spherical version of a model of localized particles in a random potential which are subject to a power-law interaction $U_{ij} \sim r_{ij}^{-\sigma}$. In the case of a repulsive interaction with $\sigma = 1$ the model is identical to the spherical version of the Coulomb-glass model of disordered localized electrons. The use of continuous variables instead of discrete occupation numbers of the sites renders the model exactly solvable. Analytic results are obtained for the free energy and for the single-particle density of states (DOS) as an example for single-particle properties. The zero-temperature DOS shows a hard gap close to the chemical potential.

I. INTRODUCTION

Many of the low-temperature properties of doped semiconductors,[1] granular metals,[2] or ionic conductors[3] are strongly influenced by the simultaneous presence of disorder and strong long-range correlations like the Coulomb interaction between localized particles. These systems have been investigated for more than 20 years;[4,5] however, we are still far from having obtained a complete picture of their physics. Not even the phase diagram, thermodynamic properties, or simple single-particle properties are known in detail.[6] In particular, an essential quantity which is still under controversial discussion is the density of states (DOS) of the single-particle excitations. It is generally accepted that the interplay between disorder and interactions decreases the DOS close to the chemical potential, causing a gap which is called the Coulomb gap. This gap is expected to be a soft gap, which means that the DOS vanishes at the chemical potential but it is finite for every energy different from the chemical potential.[1,7] However, recent numerical results about the behavior of the DOS within the gap[8] are not in agreement with theoretical predictions.[9,10] Thus further studies are necessary.

Almost all theoretical investigations of disordered interacting systems with localized states are based on the lattice model or Coulomb-glass model introduced by Efros and Shklovskii,[1] which may easily be generalized to a model with arbitrary power-law interaction:

$$H = \tfrac{1}{2} \sum_{i,j} U_{ij} n_i n_j + \sum_i \varphi_i n_i \ , \quad U_{ij} = \frac{U_0}{r_{ij}^{\sigma}} > 0 \ , \tag{1}$$

where sites i form a regular lattice in D dimensions, and r_{ij} is the distance between sites i and j. The variable $n_i = \pm\frac{1}{2}$ describes the occupation of site i. Values φ_i of the potential are independent random variables, taken from a Gaussian or box distribution with a zero mean and the variance Φ^2. In spite of the very simple structure of this Hamiltonian, the study of its properties is very complicated. In particular the usual many-body techniques are not suitable because of the long-range nature of the interaction.

In this paper, therefore, we investigate the single-particle DOS of a simpler model which is exactly solvable but retains some of the main properties of the lattice Hamiltonian (1). This simpler model is the spherical version of Hamiltonian (1). To obtain the spherical Hamiltonian the discrete occupation numbers n_i in Eq. (1) are replaced by continuous variables S_i, ranging from $-\infty$ to ∞. In the language of the spherical model the potential φ_i corresponds to a random field; that means in the following we shall treat a random-field spherical model with long-range interactions. To make the model well defined and to avoid states with diverging energy, a global constraint on the values of the variables (the spherical constraint) has to be added;

$$\sum_i S_i^2 = \frac{N}{4} \ , \tag{2}$$

where N is the total number of sites in the system. This constraint also illuminates the name of the model because the possible values of the N-dimensional S vector are located on an N-dimensional hypersphere of radius $N^{1/2}/2$. The spherical model was first introduced by Berlin and Kac[11] as an approximation of the nearest-neighbor Ising model. Later it was also used to investigate systems with power-law interactions.[12] Stanley[13] showed that the spherical model is equivalent to the n-vector model in the limit $n \to \infty$. Therefore the spherical model may serve as a starting point for a $1/n$ expansion of the properties under investigation. Hornreich and Schuster[14] and Vojta[15] studied the thermodynamic properties of the random-field spherical model with short- and long-range power-law interactions, using the replica trick to calculate the averaged free energy.

The treatment of the spherical model is much easier than that of the corresponding model with discrete variables, because the trace over the variables is a multiple integral for continuous variables while it is a multiple sum for discrete variables. Therefore it is possible to calculate exactly not only the partition function and the free energy but also single-particle properties like the single-

particle DOS. This will be shown in the present paper.

The paper is organized as follows. In Sec. II we briefly summarize the calculation of the partition function and the free energy as far as it is necessary for the determination of the single-particle DOS. In Sec. III A we study the DOS for the spherical model, while we slightly modify the model in Sec. III B by introducing a single discrete variable, because this modified model is closer to the original lattice model. Finally, some conclusions are presented in Sec. IV.

II. PARTITION FUNCTION AND FREE ENERGY

The evaluation of the average free energy of a random system is a difficult task since it involves an average over the logarithm of the partition function. The usual way to overcome this difficulty is the replica trick[16] which was also used by Hornreich and Schuster[14] and Vojta[15] in the study of the random-field spherical model. However, in the case of the random-field spherical model one can avoid the replica trick for the calculation of the free energy, as was first noticed by Shwartz[17] for a special case of the model. In this section we briefly summarize the determination of the partition function and the free energy for a general random-field spherical model without using replicas.

The partition function for the spherical version of Hamiltonian (1) for a fixed configuration of the random potential is given by

$$Z[\varphi]=\prod_i \left[\int dS_i\right] \delta\left[\frac{N}{4}-\sum_i S_i^2\right] \times e^{-(\beta/2)\sum_{i,j} U_{ij}S_iS_j-\beta\sum_i \varphi_i S_i} , \tag{3}$$

where β is the inverse temperature $\beta=(kT)^{-1}$. Here the spherical constraint (2) is included by means of the δ function. Using the Fourier representation of the δ function the Gaussian integrals over the variables S_i may be carried out. A single integral over the variable z that stems from the Fourier representation of the δ function remains;

$$Z[\varphi]=\frac{\pi^{N/2}}{2\pi i}\int dz\, e^{N\Psi(z)} , \tag{4}$$

where Ψ is given by

$$\Psi(z)=\frac{z}{4}-\frac{1}{2N}\mathrm{Tr}\ln\mathbf{V}+\frac{\beta^2}{4N}\varphi\mathbf{V}^{-1}\varphi . \tag{5}$$

The matrix $\mathbf{V}$ comprises the matrix elements $V_{ij}=z\delta_{ij}+\beta U_{ij}/2$. In the thermodynamic limit $N\to\infty$ integral (4) can be calculated by means of the saddle-point method, yielding

$$Z[\varphi]=\pi^{N/2}(2\pi N|\Psi''(z_0)|)^{-1/2}e^{N\Psi(z_0)} , \tag{6}$$

where Ψ'' denotes the second derivative of Ψ with respect to z. The saddle-point equation

$$0=\frac{1}{4}-\frac{1}{2N}\mathrm{Tr}\mathbf{V}^{-1}-\frac{\beta^2}{4N}\varphi\mathbf{V}^{-2}\varphi \tag{7}$$

determines z_0. Having calculated the saddle-point value z_0, the free energy f per site for a fixed realization of the random potentials is given by $-\beta f[\varphi]=\Psi(z_0)$.

In principle the saddle-point value z_0 and the free energy f could depend on the special realization of the random potentials. However, a detailed analysis (see Appendix A) shows that z_0 and f are *independent* of the realization of the random potentials (for almost all realizations) in the thermodynamic limit $N\to\infty$. This means that in this limit the thermodynamic quantities are the same for all samples and therefore are equal to their ensemble average. In other words, the thermodynamic quantities are self-averaging. Consequently one can simply average Eqs. (5) and (7) with respect to the random potentials φ to obtain the correct mean free energy. As mentioned in Sec. I, the random potentials are assumed to be δ correlated, so that $\langle\varphi_i\varphi_j\rangle=\Phi^2\delta_{ij}$. Then the mean free energy is given by

$$-\langle f\rangle=\frac{s_0}{4}-\frac{1}{2N\beta}\sum_{\mathbf{k}}\ln(\beta s_0+\beta U_{\mathbf{k}}/2) + \frac{\Phi^2}{4N}\sum_{\mathbf{k}}\frac{1}{s_0+U_{\mathbf{k}}/2} , \tag{8}$$

with

$$0=\frac{1}{4}-\frac{1}{2N\beta}\sum_{\mathbf{k}}\frac{1}{s_0+U_{\mathbf{k}}/2}-\frac{\Phi^2}{4N}\sum_{\mathbf{k}}\frac{1}{(s_0+U_{\mathbf{k}}/2)^2} , \tag{9}$$

where $U_{\mathbf{k}}$ denotes the Fourier transform of the interaction U_{ij}, and s_0 is defined by $z_0=\beta s_0$. We note that these results also have been obtained by means of the replica method.[14,15]

An interesting question is whether the disordered high-temperature phase of the model remains stable for decreasing temperature, or whether a phase transition occurs. A detailed analysis[15] of Eqs. (8) and (9) shows that the random-field spherical model with long-range repulsive interactions (as well as the model with short-range interactions[14]) does not show a phase transition to an ordered low-temperature phase for spatial dimensions $D\le 4$. In higher dimensions it may have a transition for weak random potential. In the following we restrict our investigations to the case where a phase transition does not occur, and the system stays disordered down to zero temperature.

III. SINGLE-PARTICLE DENSITY OF STATES

A. DOS of the spherical model

The single-particle excitation energies ϵ_i which correspond to adding or removing a particle from the system are determined by

$$\epsilon_i=\varphi_i+\sum_{j\neq i}U_{ij}S_j . \tag{10}$$

The DOS for these single-particle excitations, averaged over the random potentials, is given by

$$\langle\bar{g}(\epsilon)\rangle_\varphi=\frac{1}{N}\sum_i\langle\langle\delta(\epsilon-\epsilon_i)\rangle_T\rangle_\varphi \tag{11a}$$

or

$$\langle g(\epsilon)\rangle_\varphi = \langle \langle \delta(\epsilon-\epsilon_0)\rangle_T \rangle_\varphi , \tag{11b}$$

where $\langle\cdot\rangle_T$ and $\langle\cdot\rangle_\varphi$ are the thermal average and the average over the random potentials, respectively. Both Eqs. (11a) and (11b) yield the same result, since all sites of the lattice are equivalent after the φ average.

To investigate the DOS (11b), we start with its Fourier transform $G(t)=\int d\epsilon\, e^{it\epsilon} g(\epsilon)$, which may be represented as

$$G(t)=\frac{1}{Z[\varphi]}\prod_i \left[\int dS_i\right] \delta\left[\frac{N}{4}-\sum_i S_i^2\right] \exp\left[-\frac{\beta}{2}\sum_{i,j} U_{ij}S_iS_j-\beta\sum_i \varphi_i S_i+it\varphi_0+it\sum_{j\neq 0} U_{0j}S_j\right] . \tag{12}$$

Further calculations are completely analogous to those for the partition function. Using the Fourier representation of the δ function and carrying out the Gaussian integral over the occupation numbers S_i, one obtains a single integral which may be evaluated by means of the saddle-point method. The contribution proportional to N in the exponent of the integrand is identical to that in the partition function, and consequently the saddle-point values are identical. The partition function in the nominator completely cancels out, and the resulting expression for $G(t)$ is

$$G(t)=\exp\left[-\frac{t^2}{4N\beta}\sum_{\mathbf{k}}\frac{U_{\mathbf{k}}^2}{s_0+U_{\mathbf{k}}/2} + \frac{it}{N^{1/2}}\left[\sum_{\mathbf{k}}\varphi_{\mathbf{k}}-\tfrac{1}{2}\sum_{\mathbf{k}}\frac{\varphi_{\mathbf{k}}U_{\mathbf{k}}}{s_0+U_{\mathbf{k}}/2}\right]\right] , \tag{13}$$

where $U_{\mathbf{k}}$ and $\varphi_{\mathbf{k}}$ are the Fourier transforms of the interaction U_{ij} and the random potential φ_i, respectively. The φ average is easily carried out for a Gaussian distribution of the random potentials. After the inverse Fourier transformation one obtains the following general behavior of the averaged single-particle DOS:

$$\langle g(\epsilon)\rangle_\varphi=(2\pi h_0^2)^{-1/2}e^{-\epsilon^2/2h_0^2} . \tag{14}$$

For any form of the interaction U_{ij}, any strength of the random potentials Φ, and any temperature the DOS is a simple Gaussian. The properties of U_{ij}, Φ, and the inverse temperature β only determine the value of the constant h_0.

Obviously, the DOS does not decrease close to the chemical potential, and in contrast to the original Hamiltonian (1) there is no Coulomb gap. This difference may be understood as follows. If in the original (generalized) Coulomb glass (1) the random potential changes by an infinitesimal amount so that the thermodynamic field of a site i changes from an infinitesimal positive value to an infinitesimal negative value; then the occupation number n_i of this site changes discontinuously from $-\frac{1}{2}$ to $+\frac{1}{2}$. This flip of n_i causes a relaxation process of the neighboring sites, which alters the single-particle energy ϵ_i by a finite amount, shifting it away from the chemical potential. Consequently a gap arises. In the case of the spherical model an infinitesimal change of the field causes only an infinitesimal change of the continuous variable S_i, and this, in turn, causes only an infinitesimal relaxation of the neighborhood so that a finite gap does not arise.

B. A modified spherical model

In this section we slightly modify the random-field spherical model so that it remains exactly solvable, while some of its properties can be expected to be closer to the original model (1) with discrete variables. The discussion at the end of Sec. III A suggests making one of the S_i's (for definiteness let us say S_0) a discrete variable $S_0=\pm\frac{1}{2}$, while the other variables remain continuous. Then expressions (11a) and (11b) for the DOS no longer yield the same result. The quantity we are now interested in is the local DOS at site 0, as given by Eq. (11b), which should be closer to the DOS of the original Hamiltonian (1) than result (14) from the spherical model.

To proceed, we have to recalculate the partition function Z, taking into account the new constraint $S_0=\pm\frac{1}{2}$. We represent this constraint by a sum of two terms with respective δ functions. The partition function for a fixed configuration of the random potentials is then given by

$$Z[\varphi]=\sum_{\xi=\pm\frac{1}{2}}\prod_i\left[\int dS_i\right]\delta(\xi-S_0)\,\delta\left[\frac{N}{4}-\sum_i S_i^2\right] \times e^{-(\beta/2)\sum_{i,j}U_{ij}S_iS_j-\beta\sum_i\varphi_iS_i} . \tag{15}$$

Using the standard manipulations above to carry out the Gaussian integral over S_i, we arrive at

$$Z[\varphi]=\pi^{N/2}[2\pi N|\Psi''(z_0)|]^{-1/2}e^{N\Psi(z_0)}\sum_{\xi=\pm\frac{1}{2}}F_1(\varphi,\xi,z_0) , \tag{16}$$

where $F_1(\varphi,\xi,z_0)$ is of the order N^0 for $N\to\infty$ and therefore does not contribute to the free energy per site. $\Psi(z)$ and the saddle-point equation are again given by Eqs. (5) and (7), reflecting the expected result, namely that a modification of the model at a single site cannot change its thermodynamic behavior.

In analogy to the partition function, we have to reformulate Eq. (12) for the Fourier transform $G(t)$ of the local DOS at site 0, yielding

$$G(t)=\frac{1}{Z[\varphi]}\sum_{\xi=\pm 1/2}\prod_i\left[\int dS_i\right]\delta(\xi-S_0)\delta\left[\frac{N}{4}-\sum_i S_i^2\right]\exp\left[-\frac{\beta}{2}\sum_{i,j}U_{ij}S_iS_j-\beta\sum_i\varphi_iS_i+it\varphi_0+it\sum_{j\neq 0}U_{0j}S_j\right]. \tag{17}$$

After carrying out the Gaussian integrations over occupation numbers S_i, and applying the saddle-point approximation, the contributions from $\Psi(z_0)$, including some prefactors, cancel out. However, in contrast to the spherical model [see Eqs. (12) and (13)] the partition function does not cancel out completely. Performing the inverse Fourier transform leads to

$$g(\epsilon)=\sum_\xi(4\pi C_1)^{-1/2}e^{-(\epsilon-C_2)^2/4C_1}e^{\beta\xi h}\Big/\sum_\xi e^{\beta\xi h}. \tag{18}$$

The parameters C_1, C_2, and h depend on the inverse temperature β, the random potentials φ_i, and the value of ξ. The explicit expressions for the parameters given in Appendix B show that the random potentials are contained both in the nominator and in the denominator of the right-hand side of (18). This makes it difficult to average over the random potentials; however, the average can be performed exactly at least for zero temperature and a Gaussian distribution of the random fields. Details of this somewhat lengthy calculation are presented in Appendix B. The final expression for the averaged zero-temperature DOS is then

$$\langle g(\epsilon)\rangle_\varphi=(2\pi h_0^2)^{-1/2}[e^{-(\epsilon-\Delta)^2/2h_0^2}\theta(\epsilon-\Delta)+e^{-(\epsilon+\Delta)^2/2h_0^2}\theta(-\epsilon-\Delta)], \tag{19}$$

with

$$h_0^{-1}=\frac{1}{N}\sum_{\mathbf{k}}(s_0+U_{\mathbf{k}}/2)^{-1} \tag{20a}$$

and

$$\Delta=s_0-h_0. \tag{20b}$$

Thus the zero-temperature DOS of the modified spherical model shows a hard gap around the chemical potential (see Fig. 1) independent of the characteristics of the interaction U_{ij}. In particular, there is also a gap in the case of short-range interactions, while the original lattice Hamiltonian (1) with short-range interactions is expected not to have a gap in the DOS but only a slight decrease close to the chemical potential. This can be easily derived by means of the arguments of Efros and Shklovskii.[1]

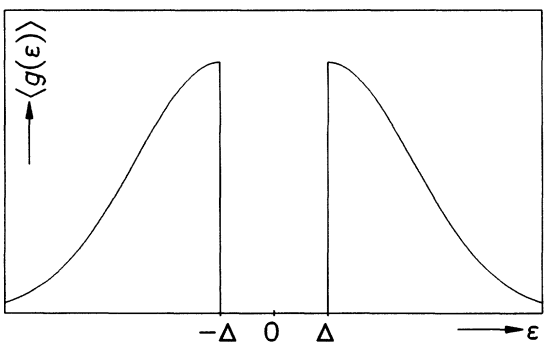

FIG. 1. Zero-temperature local single-particle density of states at site 0 for the modified spherical model.

How can this behavior be understood in comparison with the soft gap in the DOS of the original lattice Hamiltonian (1)? A simple explanation may be obtained by looking again at the relaxation process after the change of an occupation number n_i. In the original Hamiltonian a neighboring particle will relax after such a flip only with a certain probability depending on the values of the random potential and the occupation of the other sites nearby, because the hop of a particle means the flipping of two occupation numbers and thus requires a finite energy. Therefore a shift of the single-particle energy of site i due to a relaxational change of the occupation of a neighboring site arises only with a certain probability, and this results in a soft gap. In our modified spherical model the flip of S_0 always causes possibly small but finite changes of all other variables S_i because they are continuous. These add up and, in turn, always shift the single-particle energy ϵ_0 by a finite amount away from the chemical potential generating the hard gap.

In the following we want to discuss the dependence of the width Δ of the hard gap in the zero-temperature DOS on the width of the random-field distribution Φ, the interaction strength U_0, the interaction exponent σ, and the dimensionality D of the system. The calculations are restricted to the limit $U_0\ll\Phi$. The opposite limit $U_0\gg\Phi$ is much simpler; in lowest order the gap width Δ is simply the (generalized) Madelung energy of the lattice corresponding to an alternating occupation of the lattice sites, i.e., corresponding to a Wigner crystal. To calculate the gap width for $U_0\ll\Phi$ one has to solve the saddle-point Eq. (7) in the limits $\beta\to\infty$ and $\Phi\to\infty$. In lowest order in $1/\Phi$, one obtains $s_0=\Phi$. Inserting this saddle-point value into Eq. (20a) allows us to perform the calculation of h_0, which is presented in Appendix C. The resulting gap width depends on the dimension and interaction exponent via only the ratio $\alpha=D/\sigma$;

$$\Delta=s_0-h_0\sim U_0^{\alpha/(\alpha-1)}/\Phi^{1/(\alpha-1)}, \tag{21}$$

if α is larger than 2. For $\alpha\leq 2$, we get

$$\Delta=s_0-h_0\sim U_0^2/\Phi, \tag{22}$$

which also holds for short-range interactions. In the case of the Coulomb interaction $\sigma=1$ we get in particular $\Delta\sim U_0^2/\Phi$ for the two-dimensional model and

$\Delta \sim U_0^{3/2}/\Phi^{1/2}$ in three spatial dimensions. Thus the width of the Coulomb gap in our modified spherical model shows the same dependence on parameters U_0 and Φ as the width of the Coulomb gap in the original Coulomb-glass model (1), which was first calculated by Srinivasan,[5] and later also obtained by Efros and Shklovskii[1] and Hunt and Pollak.[18]

IV. CONCLUSIONS

In this paper we have investigated the single-particle DOS for both the spherical version of a lattice model of disordered and correlated localized particles and a modified spherical model. The modified model has a single discrete variable S_0, while the other variables $S_{i\neq 0}$ remain continuous. These spherical models are easier to treat than the corresponding lattice model with discrete occupation numbers. Therefore many properties including the DOS of the spherical models can be calculated exactly.

The DOS of the spherical model with only continuous variables S_i shows a Gaussian behavior. Such behavior is typical for the spherical model at temperatures above a transition to an ordered phase. It corresponds to the Gaussian distribution for the magnitudes of the variables S_i above the transition.[11] We have shown that, independent of the kind of interaction, the DOS does not decrease near the chemical potential, and in particular there is no Coulomb gap. The absence of the Coulomb gap in the spherical model may also be understood in terms of the relaxation of the particles after a change of the random potential at a single site (for definiteness let us consider the site 0). If the occupation numbers are discrete, an infinitesimal change of the potential φ_0 will lead either to a flip of the occupation number n_0 (if and only if the thermodynamic field of site 0 changes sign) or will not lead to a change of the occupation number. In the former case the surrounding particles may relax, shifting the single-particle energy away from the chemical potential. In this way a gap may arise due to the discontinuous dependence of the occupation numbers on the potential. In the spherical model, however, an infinitesimal change of the potential φ_0 will always yield an infinitesimal change of all variables S_i. Therefore the single-particle energy is shifted only by an infinitesimal amount. A finite gap does not arise in this case because the occupation numbers are continuous functions of the potential.

This behavior has led us to suggest a modification of the spherical model, allowing one of the variables S_i to become discrete (for definiteness we choose $S_0=\pm\frac{1}{2}$). For this modified model the local DOS at site 0 shows a hard gap for any kind of interaction including short-range interactions, in contrast to the original lattice model with discrete variables, in which the DOS has a soft gap for long-range interactions but does not have a gap for short-range interactions. The reason for the occurrence of the hard gap is again due to the constraint of the occupation number of site 0, which can only change discontinuously (flip from $-\frac{1}{2}$ to $\frac{1}{2}$ or vice versa). Consequently, during the relaxation process after a change of S_0 all other occupation numbers $S_{i\neq 0}$ change their values by finite amounts, and therefore the single-particle energy ϵ_0 is shifted by a finite amount and a hard gap results. In contrast, in the original lattice model (1) the occupation numbers $n_{i\neq 0}$ are discontinuous and therefore flip only with a certain probability during the relaxation process. This means that the shift of the single-particle energy occurs only with a certain probability and therefore the gap becomes soft.

Nevertheless it is interesting to analyze the width of our hard gap arising in the modified spherical model, and compare it with the width of the soft gap in the original lattice model. Here the dependence of the width of this generalized "Coulomb" gap on the parameters of the model has been derived in the limit of strong random potential. The width depends only via the ratio D/σ on the dimension D and interaction exponent σ. The results in 2D as well as in 3D are in agreement with results for the original lattice Hamiltonian.

The spherical model is equivalent to the $n\rightarrow\infty$ limit of an n-vector model. Therefore it would be interesting to take the spherical model as a starting point for a $1/n$-expansion of the quantities under consideration. However, this remains a task for the future.

APPENDIX A. SELF-AVERAGING OF THERMODYNAMIC QUANTITIES

In order to investigate whether the saddle-point value z_0 and the free energy f per site depend on the special realization of the random potentials in the thermodynamic limit, we have to study the behavior of $a(\varphi,z)=\varphi\mathbf{V}^{-1}\varphi/N$ for $N\rightarrow\infty$. a may be represented as

$$a(\varphi,z)=\frac{1}{N}\sum_{\mathbf{k}}(z+\beta U_{\mathbf{k}}/2)^{-1}\frac{1}{N}\sum_{i,j}\cos[\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)]\varphi_i\varphi_j\ . \tag{A1}$$

For δ-correlated random potentials $\langle\varphi_i\varphi_j\rangle=\Phi^2\delta_{ij}$, it is easily seen that $\langle a(\varphi,z)\rangle$ is of the order N^0 in the limit $N\rightarrow\infty$. The mean-square deviation $\langle\Delta a^2\rangle$ is given by

$$\langle\Delta a^2\rangle=\langle a^2\rangle-\langle a\rangle^2=\frac{1}{N^2}\sum_{\mathbf{k},\mathbf{k}'}(z+\beta U_{\mathbf{k}})^{-1}(z+\beta U_{\mathbf{k}'})^{-1}$$

$$\times\frac{1}{N^2}\sum_{i,j,m,n}\cos[\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)]\cos[\mathbf{k}'\cdot(\mathbf{r}_m-\mathbf{r}_n)](\langle\varphi_i\varphi_j\varphi_m\varphi_n\rangle-\langle\varphi_i\varphi_j\rangle\langle\varphi_m\varphi_n\rangle)\ . \tag{A2}$$

Unless $i=j=m=n$, we use $\langle\varphi_i\varphi_j\varphi_m\varphi_n\rangle = \langle\varphi_i\varphi_j\rangle\langle\varphi_m\varphi_n\rangle + \langle\varphi_i\varphi_m\rangle\langle\varphi_j\varphi_n\rangle + \langle\varphi_i\varphi_n\rangle\langle\varphi_j\varphi_m\rangle$ for our δ-correlated random potential, and obtain that $\langle\Delta a^2\rangle$ is of the order of $1/N$ for $N\to\infty$. Consequently the distribution of $a(\varphi,z)$ approaches a δ distribution in the thermodynamic limit. Analogous calculations can be made for the corresponding term in the saddle-point Eq. (7). Therefore the distribution of the saddle-point value z_0 also approaches a δ distribution. In summary, the saddle-point value z_0 and the free energy f per site do not depend on the special realization of the random potentials. (However, there are a few configurations for which this is not valid. These configurations form a set of measure zero and therefore do not contribute to the averaged thermodynamic quantities. For instance, the configurations $\varphi_i \equiv \text{const} \neq 0$ belong to that set.)

APPENDIX B. ZERO-TEMPERATURE DOS OF THE MODIFIED SPHERICAL MODEL

The single-particle DOS $g(\epsilon)$ for a fixed realization of the random potentials is determined by Eq. (18). The parameters C_1, C_2, and h are given by

$$
\begin{aligned}
C_1 &= \frac{1}{\beta}\left[\frac{1}{4N}\sum_{\mathbf{k}}\frac{U_{\mathbf{k}}^2}{\mu_{\mathbf{k}}} - \left[\frac{1}{2N}\sum_{\mathbf{k}}\frac{U_{\mathbf{k}}}{\mu_{\mathbf{k}}}\right]^2 \Big/ \left[\frac{1}{N}\sum_{\mathbf{k}}\frac{1}{\mu_{\mathbf{k}}}\right]\right], \\
C_2 &= \frac{1}{2N^{1/2}}\sum_{\mathbf{k}}\left[2\varphi_{\mathbf{k}} - \frac{\varphi_{\mathbf{k}}U_{\mathbf{k}}}{\mu_{\mathbf{k}}}\right] + \left[-\frac{\xi}{N}\sum_{\mathbf{k}}\frac{U_{\mathbf{k}}}{\mu_{\mathbf{k}}} + \frac{1}{2N^{3/2}}\sum_{\mathbf{k}}\frac{\varphi_{\mathbf{k}}}{\mu_{\mathbf{k}}}\sum_{\mathbf{k}}\frac{U_{\mathbf{k}}}{\mu_{\mathbf{k}}}\right] \Big/ \left[\frac{1}{N}\sum_{\mathbf{k}}\frac{1}{\mu_{\mathbf{k}}}\right], \\
h &= \left[\frac{1}{N^{1/2}}\sum_{\mathbf{k}}\frac{\varphi_{\mathbf{k}}}{\mu_{\mathbf{k}}}\right] \Big/ \left[\frac{1}{N}\sum_{\mathbf{k}}\frac{1}{\mu_{\mathbf{k}}}\right],
\end{aligned}
\tag{B1}
$$

where $\mu_{\mathbf{k}} = s_0 + U_{\mathbf{k}}/2$. Using the definitions of h_0 and Δ [see Eq. (20)], the expressions for C_1 and C_2 can be simplified to

$$
\begin{aligned}
C_1 &= \Delta/\beta, \\
C_2 &= h + 2\xi\Delta.
\end{aligned}
\tag{B2}
$$

Then the DOS $g(\epsilon)$ may be written as

$$
g(\epsilon) = \sum_{\xi}(4\pi\Delta/\beta)^{-1/2} e^{-\beta(\epsilon-h-2\xi\Delta)^2/4\Delta} e^{\beta\xi h} \Big/ \sum_{\xi} e^{\beta\xi h}. \tag{B3}
$$

Here the random potentials φ_i are only contained in the thermodynamic field h of site 0. The averaged DOS $\langle g(\epsilon)\rangle_{\varphi}$ can be obtained from Eq. (B3) simply by averaging with respect to h, which has a Gaussian probability distribution of the width

$$
\langle h^2\rangle_{\varphi} = h_0^2(1 - 2/\beta h_0). \tag{B4}
$$

For zero temperature we obtain $C_1 = 0$, and the DOS is an average over δ functions

$$
\langle g(\epsilon)\rangle_{\varphi} = \left\langle \sum_{\xi}\delta(\epsilon - h - 2\xi\Delta)\theta(\xi h)\right\rangle_h, \tag{B5}
$$

where θ is the Heaviside function. Carrying out the average yields the DOS given in Eq. (19).

APPENDIX C. CALCULATION OF THE GAP WIDTH FOR $\Phi \gg U_0$

In order to calculate the gap width Δ [Eq. (20b)], we first have to solve the saddle-point Eq. (9). For zero temperature and in lowest order of U_0/Φ, Eq. (9) yields $s_0 = \Phi$. If $U_{\mathbf{k}}$ is finite for all $\mathbf{k}$, then h_0 may be calculated easily by expanding the right-hand side of Eq. (20a) in powers of s_0^{-1}. This yields

$$
h_0^{-1} = s_0^{-1} + cU_0^2 s_0^{-3} + \cdots. \tag{C1}
$$

For long-range interactions with $D > \sigma$, however, $U_{\mathbf{k}}$ diverges for $\mathbf{k}\to\mathbf{0}$ as $|\mathbf{k}|^{\sigma-D}$ as calculated by Nijboer and de Wette.[19] Therefore $s_0 + U_{\mathbf{k}}/2$ can be expanded for all $\mathbf{k}$ except for $\mathbf{k}$ values in a small region around $\mathbf{k}=\mathbf{0}$. This region is integrated separately. In summary, for long-range interactions with $\alpha = D/\sigma > 1$ we have

$$
h_0^{-1} = s_0^{-1} + bU_0^{\alpha/(\alpha-1)} s_0^{-(2\alpha-1)/(\alpha-1)} + cU_0^2 s_0^{-3} + \cdots. \tag{C2}
$$

For $\alpha \le 2$ the leading terms are nevertheless given by Eq. (C1). For $\alpha > 2$ the leading terms of the expansion are the first two terms in Eq. (C2). Inserting $s_0 = \Phi$ and Eqs. (C1) or (C2) into Eq. (20b), we obtain Eqs. (22) and (21), respectively.

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